USN | | | | | | | | | | **06MAT31**

Third Semester B.E. Degree Examination, June 2012
Engineering Mathematics – III

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Find the Fourier series of the function,
$f(x)=\begin{cases}2-x, & 0\le x\le 4\\ x-6, & 4\le x\le 8\end{cases}$. Hence deduce that $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+....$ **(07 Marks)**

b. Find the half range cosine series for, $f(x)=x(\pi-x)$ in $0<x<\pi$. **(06 Marks)**

c. Analyse harmonically the data given below and express 'y' in Fourier series upto the second harmonics: **(07 Marks)**

x°	0	60	120	180	240	300	360
y	1.0	1.4	1.9	1.7	1.5	1.2	1.0

2 a. Find the complex Fourier transform of f(x) where
$f(x)=\begin{cases}1-x^2, & |x|<1\\ 0, & |x|\ge 1\end{cases}$. Hence find the value of $\int_0^\infty \frac{x\cos x-\sin x}{x^3}dx$. **(07 Marks)**

b. Find the Fourier cosine transform of the function,
$f(x)=\begin{cases}4x, & 0<x<1\\ 4-x, & 1<x<4\\ 0, & x>4\end{cases}$ **(06 Marks)**

c. Find the complex Fourier transform of $e^{-a^2x^2}$, a>0. Hence deduce that $e^{-x^2/2}$ is self reciprocal under the complex Fourier transform. **(07 Marks)**

3 a. Find the general solution of, $x^2(y-z)p+y^2(z-x)q=z^2(x-y)$. **(07 Marks)**

b. Form the partial differential equation by eliminating the arbitrary functions from, $z=f(x+ct)+g(x-ct)$. **(06 Marks)**

c. Solve $\frac{\partial^2 z}{\partial x^2}-2\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=0$ by the method of separation of variables. **(07 Marks)**

4 a. Derive one-dimensional heat equation in standard form. **(07 Marks)**

b. Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ for its various possible solutions, by the method of separation of variables. **(06 Marks)**

c. A string is stretched tightly between two points at a distance 'l' apart. The motion of the string is started by displacing the string into the form $u=u_0\sin\left(\frac{\pi x}{l}\right)$, from which it is released from rest. Find the displacement u(x, t) of any point at a distance x from one end at any time t. **(07 Marks)**

PART – B

5 a. Using Regula-Falsi method, find a root of $x^6 - x^4 - x^3 - 1 = 0$ in (1, 2), correct to four decimal places. Carryout three iterations. **(07 Marks)**

b. Apply Gauss-Seidel iterative method, to solve $x + 2y + 5z = 20$; $5x + 2y + z = 12$; $x + 4y + 2z = 15$. **(06 Marks)**

c. Using power method, find the largest eigen value and the corresponding eigen vector of the matrix $\begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$, starting with the initial eigen vector $(1 \quad 0 \quad 0)^T$. **(07 Marks)**

6 a. Find the values of f(38) and f(85) using suitable interpolation formulae, given **(07 Marks)**

x:	40	50	60	70	80	90
y=f(x):	184	204	226	250	276	304

b. Evaluate $\int_0^1 \sqrt{\sin x + \cos x}\, dx$ correct to two decimal places using Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule taking seven equidistant ordinates. **(06 Marks)**

c. Fit an interpolating polynomial for the data:

x:	0	1	4	8	10
y = f(x):	-5	-14	-125	-21	355

using Newton's general interpolation formula. Hence find f(2). **(07 Marks)**

7 a. Obtain Euler's equation for the variational problem in the form:

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0.$$

Modify this equation when f is independent of y. **(07 Marks)**

b. Define a geodesic on a surface. Prove that the geodesics on a plane are straight lines. **(06 Marks)**

c. Solve the variational problem $\delta\int_0^{\frac{\pi}{2}} (y^2 - y'^2)\, dx$ under the conditions y(0)=0, $y\left(\frac{\pi}{2}\right) = 2$. **(07 Marks)**

8 a. Find the z-transforms of, i) $\sin n\theta$ ii) $\cos n\theta$. **(07 Marks)**

b. Find the inverse z-transform of, $\frac{2z^2 + 3z}{(z+2)(z-4)}$. **(06 Marks)**

c. Using z-transforms, solve $y_{n+2} - 5y_{n+1} + 6y_n = 1$ with $y_0 = 0$ and $y_1 = 1$. **(07 Marks)**

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